SL Paper 1

A function is represented by the equation

$$f(x) = ax^2 + \frac{4}{x} - 3$$

a. Find $f'(x)$. [3]

b. The function $f(x)$ has a local maximum at the point where $x = -1$. [3]

Find the value of *a*.

Markscheme

a. $f(x) = ax^2 + 4x^{-1} - 3$

$f'(x) = 2ax - 4x^{-2}$ ***(A3)***

(A1) *for 2ax,* ***(A1)*** *for –4x* $^{-2}$ *and* ***(A1)*** *for derivative of –3 being zero.* ***(C3)***

[3 marks]

b. $2ax - 4x^{-2} = 0$ ***(M1)***

$2a(-1) - 4(-1)^{-2} = 0$ ***(M1)***

$-2a - 4 = 0$

$a = -2$ ***(A1)*****(ft)**

(M1) *for setting derivative function equal to 0.* ***(M1)*** *for inserting* $x = -1$ *but do not award* ***(M0)(M1)*** ***(C3)***

[3 marks]

Examiners report

a. (a) Many candidates gave up at this point. Those who attempted the derivative did so with varying success. Many could not differentiate a term with a negative index.

b. (b) In part (b) most substituted the -1 into the original function rather than the differentiated one. They did not realize they had to put the differentiated function equal to zero.

The function $f(x)$ is such that $f'(x) < 0$ for $1 < x < 4$. At the point $\mathrm{P}(4,\,2)$ on the graph of $f(x)$ the gradient is zero.

a. Write down the equation of the tangent to the graph of $f(x)$ at P. [2]

b. State whether $f(4)$ is greater than, equal to or less than $f(2)$. [2]

c. Given that $f(x)$ is increasing for $4 \leqslant x < 7$, what can you say about the point P? [2]

Markscheme

a. $y = 2$. ***(A1)(A1)*** ***(C2)***

Note: Award ***(A1)*** for $y = \ldots$, ***(A1)*** for 2.
Accept $f(x) = 2$ and $y = 0x + 2$

b. Less (than). ***(A2)*** ***(C2)***

[2 marks]

c. Local minimum (*accept minimum, smallest or equivalent*) ***(A2)*** ***(C2)***

Note: Award ***(A1)*** for stationary or turning point mentioned.
No mark is awarded for $\text{gradient} = 0$ as this is given in the question.

Examiners report

a. This question was poorly answered by many of the candidates. They could not write down the equation of the tangent, they could not say whether one value was greater or less than another and they could not answer that P was a minimum point. Most attempted the question so it was not a case that the paper was too long. This was a very good discriminator for the paper.

b. This question was poorly answered by many of the candidates. They could not write down the equation of the tangent, they could not say whether one value was greater or less than another and they could not answer that P was a minimum point. Most attempted the question so it was not a case that the paper was too long. This was a very good discriminator for the paper.

c. This question was poorly answered by many of the candidates. They could not write down the equation of the tangent, they could not say whether one value was greater or less than another and they could not answer that P was a minimum point. Most attempted the question so it was not a case that the paper was too long. This was a very good discriminator for the paper.

Consider the function $f\left(x\right) = \frac{x^4}{4}$.

a. Find *f'*(*x*) [1]

b. Find the gradient of the graph of *f* at $x = -\frac{1}{2}$. [2]

c. Find the *x*-coordinate of the point at which the **normal** to the graph of *f* has gradient $-\frac{1}{8}$. [3]

Markscheme

a. x^3 ***(A1) (C1)***

Note: Award ***(A0)*** for $\frac{4x^3}{4}$ and not simplified to x^3.

[1 mark]

b. $\left(-\frac{1}{2}\right)^3$ ***(M1)***

Note: Award ***(M1)*** for correct substitution of $-\frac{1}{2}$ into their derivative.

$-\frac{1}{8}$ (–0.125) ***(A1)(ft) (C2)***

Note: Follow through from their part (a).

[2 marks]

c. $x^3 = 8$ ***(A1)(M1)***

Note: Award ***(A1)*** for 8 seen maybe seen as part of an equation $y = 8x + c$, ***(M1)*** for equating their derivative to 8.

$(x =)\ 2$ ***(A1) (C3)***

Note: Do not accept (2, 4).

[3 marks]

Examiners report

a. [N/A]
b. [N/A]
c. [N/A]

Consider the curve $y = x^2$.

a. Write down $\frac{\mathrm{d}y}{\mathrm{d}x}$. [1]

b. The point $\mathrm{P}(3,\ 9)$ lies on the curve $y = x^2$. Find the gradient of the tangent to the curve at P . [2]

c. The point $\mathrm{P}(3,\ 9)$ lies on the curve $y = x^2$. Find the equation of the normal to the curve at P . Give your answer in the form $y = mx + c$. [3]

Markscheme

a. $2x$ ***(A1) (C1)***

b. 2×3 ***(M1)***

$= 6$ ***(A1) (C2)***

c. $m(\mathrm{perp}) = -\frac{1}{6}$ ***(A1)(ft)***

Note: Follow through from their answer to part (b).

Equation $(y-9) = -\frac{1}{6}(x-3)$ ***(M1)***

Note: Award ***(M1)*** for correct substitution in any formula for equation of a line.

$y = -\frac{1}{6}x + 9\frac{1}{2}$ ***(A1)*(ft)** ***(C3)***

Note: Follow through from correct substitution of their gradient of the normal.
Note: There are no extra marks awarded for rearranging the equation to the form $y = mx + c$.

Examiners report

a. [N/A]
b. [N/A]
c. [N/A]

The equation of a curve is given as $y = 2x^2 - 5x + 4$.

a. Find $\frac{\mathrm{d}y}{\mathrm{d}x}$. [2]

b. The equation of the line *L* is $6x + 2y = -1$. [4]

Find the *x*-coordinate of the point on the curve $y = 2x^2 - 5x + 4$ where the tangent is parallel to *L*.

Markscheme

a. $\frac{\mathrm{d}y}{\mathrm{d}x} = 4x - 5$ ***(A1)(A1)*** ***(C2)***

Notes: Award ***(A1)*** for each correct term. Award ***(A1)(A0)*** if any other terms are given.

[2 marks]

b. $y = -3x - \frac{1}{2}$ ***(M1)***

Note: Award ***(M1)*** for rearrangement of equation

gradient of line is –3 ***(A1)***

$4x - 5 = -3$ ***(M1)***

Notes: Award ***(M1)*** for equating their gradient to their derivative from part (a). If $4x - 5 = -3$ is seen with no working award ***(M1)(A1)(M1)***.

$x = \frac{1}{2}$ ***(A1)*(ft)** ***(C4)***

Note: Follow through from their part (a). If answer is given as (0.5, 2) with no working award the final ***(A1)*** only.

[4 marks]

Examiners report

a. The derivative of the function was correctly found by most candidates. Rearranging the equation of the line to find the gradient was also successfully performed. Most candidates could not find the *x*-coordinate of the point on the curve whose tangent was parallel to a given line. To most candidates, part (b) appeared to be disconnected to part (a).

b. The derivative of the function was correctly found by most candidates. Rearranging the equation of the line to find the gradient was also successfully performed. Most candidates could not find the *x*-coordinate of the point on the curve whose tangent was parallel to a given line. To most candidates, part (b) appeared to be disconnected to part (a).

Consider the function $f(x) = ax^3 - 3x + 5$, where $a \neq 0$.

a. Find $f'(x)$. [2]

b. Write down the value of $f'(0)$. [1]

c. The function has a local maximum at *x* = −2. [3]

Calculate the value of *a*.

Markscheme

a. $f'(x) = 3ax^2 - 3$ ***(A1)(A1)*** ***(C2)***

Note: Award a maximum of ***(A1)(A0)*** if any extra terms are seen.

b. −3 ***(A1)*(ft)** ***(C1)***

Note: Follow through from their part (a).

c. $f'(x) = 0$ ***(M1)***

Note: This may be implied from line below.

$3a(-2)^2 - 3 = 0$ ***(M1)***

$(a =)\frac{1}{4}$ ***(A1)*(ft)** ***(C3)***

Note: Follow through from their part (a).

Examiners report

a. Many candidates could find the derivative of the cubic function and find the value of the derivative at $x = 0$. For part (c) many candidates calculated the value of the function rather than the derivative at $x = -2$.

b. Many candidates could find the derivative of the cubic function and find the value of the derivative at $x = 0$.

c. Many candidates could find the derivative of the cubic function and find the value of the derivative at $x = 0$. For part (c) many candidates calculated the value of the function rather than the derivative at $x = -2$. However only the best realized that the derivative is zero at the maximum and so calculated the value of a.

Let $f(x) = 2x^2 + x - 6$

a. Find $f'(x)$. [3]

b. Find the value of $f'(-3)$. [1]

c. Find the value of x for which $f'(x) = 0$. [2]

Markscheme

a. $f'(x) = 4x + 1$ ***(A1)(A1)(A1)*** ***(C3)***

Note: Award ***(A1)*** for each term differentiated correctly.

Award at most ***(A1)(A1)(A0)*** if any extra terms seen.

[3 marks]

b. $f'(-3) = -11$ ***(A1)*(ft)** ***(C1)***

[1 mark]

c. $4x + 1 = 0$ ***(M1)***

$x = -\frac{1}{4}$ ***(A1)*(ft)** ***(C2)***

[2 marks]

Examiners report

a. This was a fairly standard question. However, some candidates found $f(-3)$ instead of $f'(-3)$. Quite a few candidates were unable to answer part (c) as they tried to find $f'(0)$ instead of finding x when $f'(x) = 0$.

b. This was a fairly standard question. However, some candidates found $f(-3)$ instead of $f'(-3)$. Quite a few candidates were unable to answer part (c) as they tried to find $f'(0)$ instead of finding x when $f'(x) = 0$.

c. This was a fairly standard question. However, some candidates found $f(-3)$ instead of $f'(-3)$. Quite a few candidates were unable to answer part (c) as they tried to find $f'(0)$ instead of finding x when $f'(x) = 0$.

The coordinates of point A are $(6,\ -7)$ and the coordinates of point B are $(-6,\ 2)$. Point M is the midpoint of AB.

L_1 is the line through A and B.

The line L_2 is perpendicular to L_1 and passes through M.

a. Find the coordinates of M. [2]

b. Find the gradient of L_1. [2]

c.i. Write down the gradient of L_2. [1]

c.ii. Write down, in the form $y = mx + c$, the equation of L_2. [1]

Markscheme

a. $(0,\ 2.5)$ **OR** $\left(0,\ -\frac{5}{2}\right)$ ***(A1)(A1)*** ***(C2)***

Note: Award ***(A1)*** for 0 and ***(A1)*** for –2.5 written as a coordinate pair. Award at most ***(A1)(A0)*** if brackets are missing. Accept "$x = 0$ and $y = -2.5$".

[2 marks]

b. $\frac{2-(-7)}{-6-6}$ ***(M1)***

Note: Award ***(M1)*** for correct substitution into gradient formula.

$= -\frac{3}{4}\ (-0.75)$ ***(A1)*** ***(C2)***

[2 marks]

c.i. $\frac{4}{3}\ (1.33333\ldots)$ ***(A1)*(ft)** ***(C1)***

Note: Award ***(A0)*** for $\frac{1}{0.75}$. Follow through from part (b).

[1 mark]

c.ii. $y = \frac{4}{3}x - \frac{5}{2}\ (y = 1.33\ldots x - 2.5)$ ***(A1)*(ft)** ***(C1)***

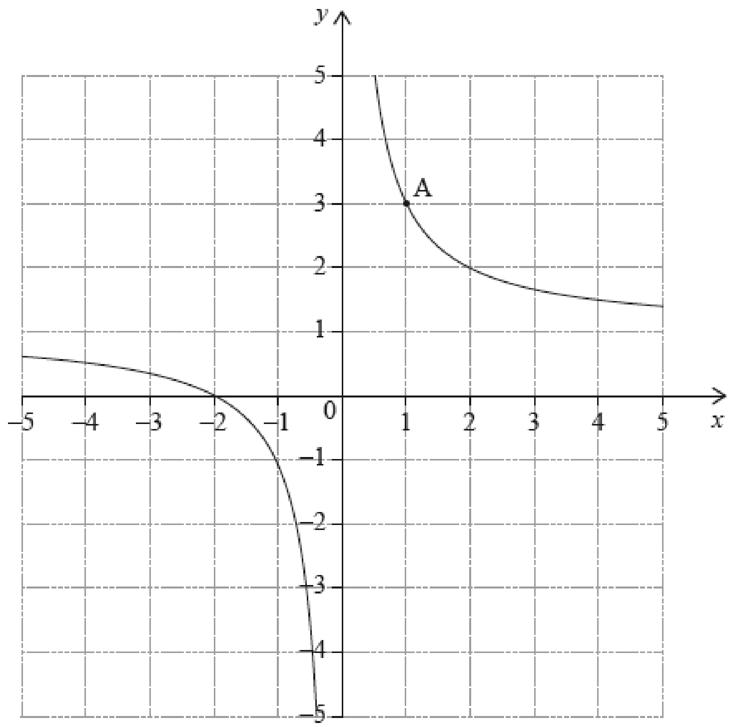
Note: Follow through from parts (c)(i) and (a). Award ***(A0)*** if final answer is not written in the form $y = mx + c$.

[1 mark]

Examiners report

a. [N/A]
b. [N/A]
c.i. [N/A]
c.ii. [N/A]

The diagram shows part of the graph of a function $y = f(x)$. The graph passes through point $\mathrm{A}(1,\ 3)$.



The tangent to the graph of $y = f(x)$ at A has equation $y = -2x + 5$. Let N be the normal to the graph of $y = f(x)$ at A.

a. Write down the value of $f(1)$. [1]

b. Find the equation of N. Give your answer in the form $ax + by + d = 0$ where $a,\ b,\ d \in \mathbb{Z}$. [3]

c. Draw the line N on the diagram above. [2]

Markscheme

a. 3 ***(A1) (C1)***

Notes: Accept $y = 3$

[1 mark]

b. $3 = 0.5(1) + c$ **OR** $y - 3 = 0.5(x - 1)$ ***(A1)(A1)***

Note: Award ***(A1)*** for correct gradient, ***(A1)*** for correct substitution of $\mathrm{A}(1,\ 3)$ in the equation of line.

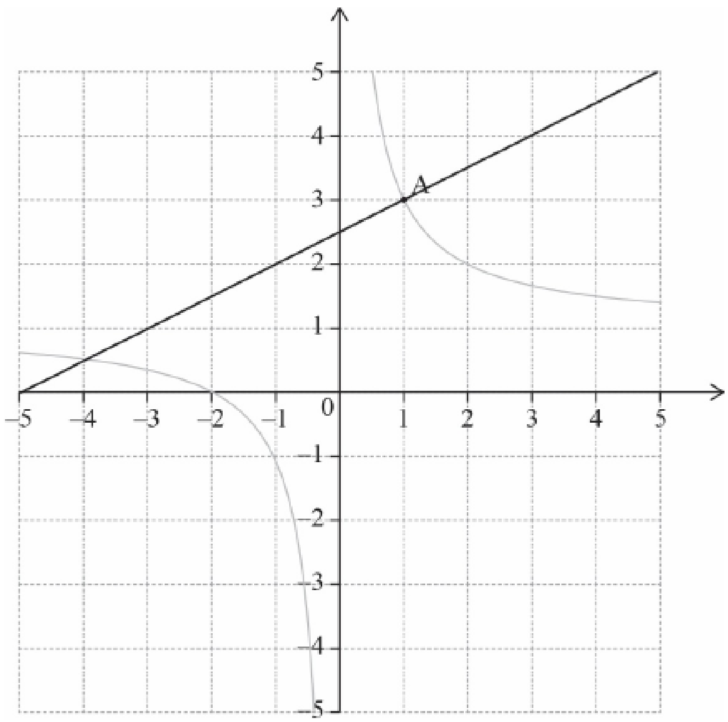
$x - 2y + 5 = 0$ or any integer multiple ***(A1)*(ft)** ***(C3)***

Note: Award ***(A1)*(ft)** for their equation correctly rearranged in the indicated form.

The candidate's answer **must** be an equation for this mark.

[3 marks]

c.



(M1)(A1)*(ft)** ***(C2)

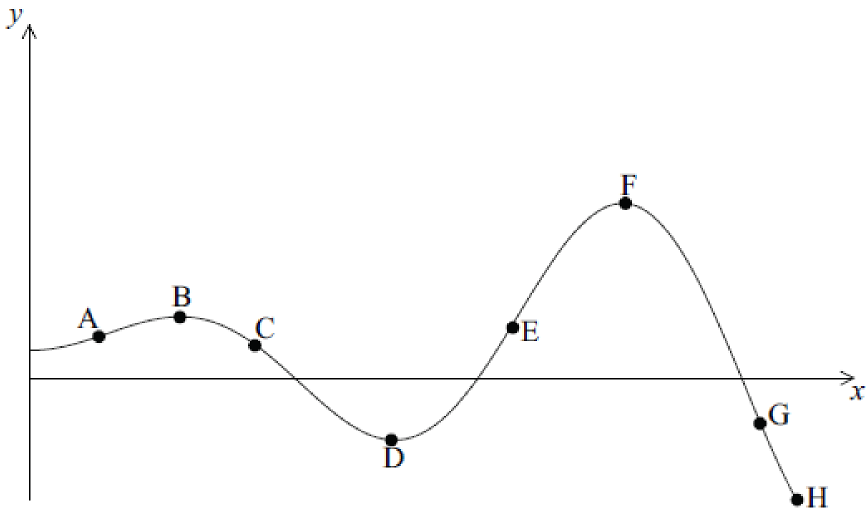
Note: Award ***M1)*** for a straight line, with positive gradient, passing through $(1,\ 3)$, ***(A1)*(ft)** for line (or extension of their line) passing approximately through 2.5 or their intercept with the y-axis.

[2 marks]

Examiners report

a. [N/A]
b. [N/A]
c. [N/A]

Consider the graph of the function $y = f(x)$ defined below.



Write down **all** the labelled points on the curve

a. that are local maximum points; [1]

b. where the function attains its least value; [1]

c. where the function attains its greatest value; [1]

d. where the gradient of the tangent to the curve is positive; [1]

e. where $f(x) > 0$ and $f'(x) < 0$. [2]

Markscheme

a. B, F ***(C1)***

b. H ***(C1)***

c. F ***(C1)***

d. A, E ***(C1)***

e. C ***(C2)***

Examiners report

a. [N/A]
b. [N/A]
c. [N/A]
d. [N/A]
e. [N/A]

Consider the function $f(x) = \frac{1}{2}x^3 - 2x^2 + 3$.

a. Find $f'(x)$. [2]

b. Find $f''(x)$. [2]

c. Find the equation of the tangent to the curve of f at the point $(1,\ 1.5)$. [2]

Markscheme

a. $\frac{3x^2}{2} - 4x$ ***(A1)(A1)*** ***(C2)***

Note: Award ***(A1)*** for each correct term and no extra terms; award ***(A1)(A0)*** for both terms correct and extra terms; ***(A0)*** otherwise.

[2 marks]

b. $3x - 4$ ***(A1)*(ft)*(A1)*(ft)** ***(C2)***

Note: accept $3x^1 - 4^0$

[2 marks]

c. $y = -2.5x + 4$ or equivalent ***(A1)*(ft)*(A1)*** ***(C2)***

Note: Award ***(A1)*(ft)** on their (a) for $-2.5x$ (must have x), ***(A1)*** for 4 or equivalent correct answer only.
Accept $y - 1.5 = -2.5(x - 1)$

[2 marks]

Examiners report

a. The final part of this question was not well answered. Most candidates could gain 4 marks in this question as most knew how to differentiate and they were required to do it twice. However, few realized that they could find the gradient of the tangent from their answer to part (a). This part was badly answered by most candidates.

b. The final part of this question was not well answered. Most candidates could gain 4 marks in this question as most knew how to differentiate and they were required to do it twice. However, few realized that they could find the gradient of the tangent from their answer to part (a). This part was badly answered by most candidates.

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A cuboid has a rectangular base of width x cm and length $2x$ cm . The height of the cuboid is h cm . The total length of the edges of the cuboid is 72 cm.

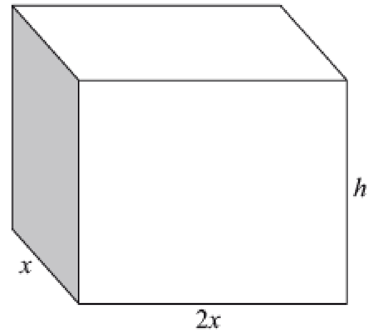


diagram not to scale

The volume, V, of the cuboid can be expressed as $V = ax^2 - 6x^3$.

a. Find the value of a. [3]

b. Find the value of x that makes the volume a maximum. [3]

Markscheme

a. $72 = 12x + 4h$ (or equivalent) ***(M1)***

Note: Award ***(M1)*** for a correct equation obtained from the total length of the edges.

$V = 2x^2(18 - 3x)$ ***(A1)***

$(a =)\ 36$ ***(A1)*** ***(C3)***

b. $\frac{\mathrm{d}V}{\mathrm{d}x} = 72x - 18x^2$ ***(A1)***

$72x - 18x^2 = 0$ **OR** $\frac{\mathrm{d}V}{\mathrm{d}x} = 0$ ***(M1)***

Notes: Award ***(A1)*** for $-18x^2$ seen. Award ***(M1)*** for equating derivative to zero.

$(x =)\ 4$ ***(A1)*(ft)** ***(C3)***

Note: Follow through from part (a).

OR

Sketch of V with visible maximum ***(M1)***

Sketch with $x \geqslant 0,\ V \geqslant 0$ and indication of maximum (e.g. coordinates) ***(A1)*(ft)**

$(x =)\ 4$ ***(A1)*(ft)** ***(C3)***

Notes: Follow through from part (a).

Award ***(M1)(A1)(A0)*** for $(4,\ 192)$.

Award ***(C3)*** for $x = 4,\ y = 192$.

Examiners report

a. The model in this question seemed to be too difficult for the vast majority of the candidates, and therefore was a strong discriminator between grade 6 and grade 7 candidates. An attempt to find an equation for the volume of the cube often started with *V* = *x* x 2*x* x *h* . Many struggled to translate the total length of the edges into a correct equation, and consequently were unable to substitute *h*. Some tried to write *x* in terms of *h* and got lost, others tried to work backwards from the expression given in the question.

b. As very few found a value for *a*, often part (b) was not attempted. When a derivative was calculated this was usually done correctly.

The point A has coordinates (4 , –8) and the point B has coordinates (–2 , 4).

The point D has coordinates (–3 , 1).

a. Write down the coordinates of C, the midpoint of line segment AB. [2]

b. Find the gradient of the line DC. [2]

c. Find the equation of the line DC. Write your answer in the form *ax* + *by* + *d* = 0 where *a* , *b* and *d* are integers. [2]

Markscheme

a. (1, –2) ***(A1)(A1) (C2)***

Note: Award ***(A1)*** for 1 and ***(A1)*** for –2, seen as a coordinate pair.

Accept *x* = 1, *y* = –2. Award ***(A1)(A0)*** if *x* and *y* coordinates are reversed.

[2 marks]

b. $\frac{1-(-2)}{-3-1}$ ***(M1)***

Note: Award ***(M1)*** for correct substitution, of their part (a), into gradient formula.

$= -\frac{3}{4}\ (-0.75)$ ***(A1)*(ft)** ***(C2)***

Note: Follow through from part (a).

[2 marks]

c. $y - 1 = -\frac{3}{4}(x+3)$ ***OR*** $y + 2 = -\frac{3}{4}(x-1)$ ***OR*** $y = -\frac{3}{4}x - \frac{5}{4}$ ***(M1)***

Note: Award ***(M1)*** for correct substitution of their part (b) and a given point.

OR

$1 = -\frac{3}{4} \times -3 + c$ ***OR*** $-2 = -\frac{3}{4} \times 1 + c$ ***(M1)***

Note: Award ***(M1)*** for correct substitution of their part (b) and a given point.

$3x + 4y + 5 = 0$ (accept any integer multiple, including negative multiples) ***(A1)*(ft)** ***(C2)***

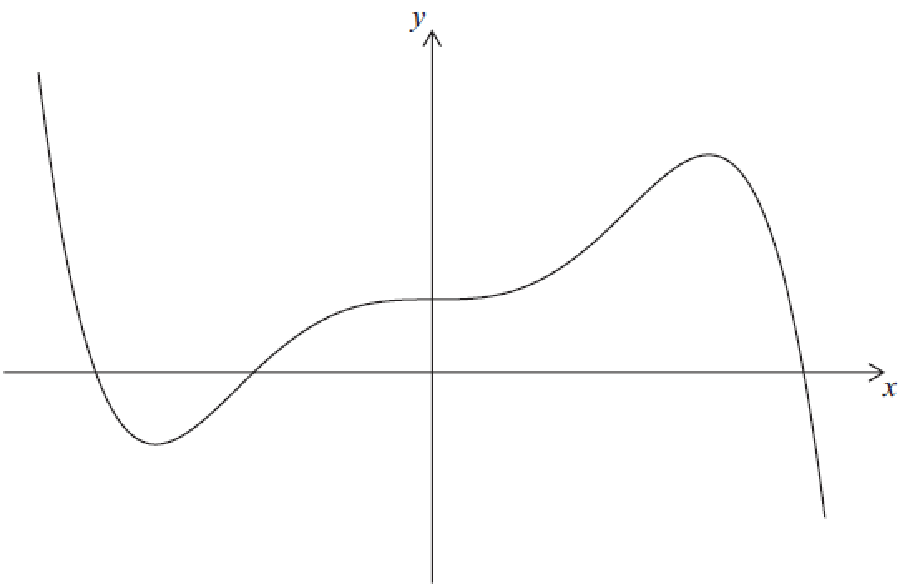
Note: Follow through from parts (a) and (b). Where the gradient in part (b) is found to be $\frac{5}{0}$, award at most ***(M1)(A0)*** for either $x = -3$ or $x + 3 = 0$.

[2 marks]

Examiners report

a. [N/A]
b. [N/A]
c. [N/A]

A sketch of the function $f(x) = 5x^3 - 3x^5 + 1$ is shown for $-1.5 \leqslant x \leqslant 1.5$ and $-6 \leqslant y \leqslant 6$.



a. Write down $f'(x)$. [2]

b. Find the equation of the tangent to the graph of $y = f(x)$ at $(1, 3)$. [2]

c. Write down the coordinates of the second point where this tangent intersects the graph of $y = f(x)$. [2]

Markscheme

a. $f'(x) = 15x^2 - 15x^4$ ***(A1)(A1)*** ***(C2)***

Note: Award a maximum of ***(A1)(A0)*** if extra terms seen.

b. $f'(1) = 0$ ***(M1)***

Note: Award ***(M1)*** for $f'(x) = 0$.

$y = 3$ ***(A1)*(ft)** ***(C2)***

Note: Follow through from their answer to part (a).

c. $(-1.38, 3)$ $(-1.38481\ldots, 3)$ ***(A1)*(ft)*(A1)*(ft)** ***(C2)***

Note: Follow through from their answer to parts (a) and (b).

Note: Accept $x = -1.38, y = 3$ ($x = -1.38481\ldots, y = 3$) .

Examiners report

a. [N/A]
b. [N/A]
c. [N/A]

A quadratic function f is given by $f(x) = ax^2 + bx + c$. The points $(0,\ 5)$ and $(-4,\ 5)$ lie on the graph of $y = f(x)$.

The y-coordinate of the minimum of the graph is 3.

a. Find the equation of the axis of symmetry of the graph of $y = f(x)$. [2]

b. Write down the value of c. [1]

c. Find the value of a and of b. [3]

Markscheme

a. $x = -2$ ***(A1)(A1)*** ***(C2)***

Note: Award ***(A1)*** for $x =$ (a constant) and ***(A1)*** for -2.

[2 marks]

b. $(c =)\ 5$ ***(A1)*** ***(C1)***

[1 mark]

c. $-\frac{b}{2a} = -2$

$a(-2)^2 - 2b + 5 = 3$ or equivalent

$a(-4)^2 - 4b + 5 = 5$ or equivalent

$2a(-2) + b = 0$ or equivalent ***(M1)***

Note: Award ***(M1)*** for two of the above equations.

$a = 0.5$ ***(A1)*(ft)**

$b = 2$ ***(A1)*(ft)** ***(C3)***

Note: Award at most ***(M1)(A1)*(ft)*(A0)*** if the answers are reversed.

Follow through from parts (a) and (b).

[3 marks]

Examiners report

a. [N/A]
b. [N/A]
c. [N/A]

Consider the curve $y = x^2 + \frac{a}{x} - 1,\ x \neq 0.$

a. Find $\frac{\mathrm{d}y}{\mathrm{d}x}$. [3]

b. The gradient of the tangent to the curve is -14 when $x = 1$. [3]

Find the value of a.

Markscheme

a. $2x - \frac{a}{x^2}$ ***(A1)(A1)(A1)*** ***(C3)***

Notes: Award ***(A1)*** for $2x$, ***(A1)*** for $-a$ and ***(A1)*** for x^{-2}.

Award at most ***(A1)(A1)(A0)*** if extra terms are present.

b. $2(1) - \frac{a}{1^2} = -14$ ***(M1)(M1)***

Note: Award ***(M1)*** for substituting 1 into their gradient function, ***(M1)*** for equating their **gradient** function to -14.

Award ***(M0)(M0)(A0)*** if the original function is used instead of the gradient function.

$a = 16$ ***(A1)*(ft)** ***(C3)***

Note: Follow through from their gradient function from part (a).

Examiners report

a. [N/A]
b. [N/A]

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a. The equation of line L_1 is $y = 2.5x + k$. Point $\mathrm{A}\ (3,\ -2)$ lies on L_1. [2]

Find the value of k.

b. The line L_2 is perpendicular to L_1 and intersects L_1 at point A. [1]

Write down the gradient of L_2.

c. Find the equation of L_2. Give your answer in the form $y = mx + c$. [2]

d. Write your answer to part (c) in the form $ax + by + d = 0$ where a, b and $d \in \mathbb{Z}$. [1]

Markscheme

a. $-2 = 2.5 \times 3 + k$ ***(M1)***

Note: Award ***(M1)*** for correct substitution of $(3,\ -2)$ into equation of L_1.

$(k =) - 9.5$ ***(A1) (C2)***

b. $-0.4\ \left(-\frac{2}{5}\right)$ ***(A1) (C1)***

c. $y - (-2) = -0.4\,(x - 3)$ ***(M1)***

OR

$-2 = -0.4\,(3) + c$ ***(M1)***

Note: Award ***(M1)*** for their gradient and given point substituted into equation of a straight line. Follow through from part (b).

$y = -0.4x - 0.8$ $\left(y = -\frac{2}{5}x - \frac{4}{5}\right)$ ***(A1)*(ft)** ***(C2)***

d. $2x + 5y + 4 = 0$ (or any integer multiple) ***(A1)*(ft)** ***(C1)***

Note: Follow through from part (c).

Examiners report

a. Question 7: Perpendicular Line

The response to this question was mixed.

Part (a) was well attempted by the majority.

b. In part (b), the gradient was not fully calculated (being left as a reciprocal) by a large number of candidates.

c. In part (c), the common error was the use of c from part (a) in the line.

d. In part (d), the notation for integer was not understood by a large number of candidates.

A small manufacturing company makes and sells x machines each month. The monthly cost C , in dollars, of making x machines is given by

$$C(x) = 2600 + 0.4x^2.$$

The monthly income I , in dollars, obtained by selling x machines is given by

$$I(x) = 150x - 0.6x^2.$$

$P(x)$ is the monthly profit obtained by selling x machines.

a. Find $P(x)$. [2]

b. Find the number of machines that should be made and sold each month to maximize $P(x)$. [2]

c. Use your answer to part (b) to find the selling price of **each machine** in order to maximize $P(x)$. [2]

Markscheme

a. $P(x) = I(x) - C(x)$ ***(M1)***

$= -x^2 + 150x - 2600$ ***(A1)*** ***(C2)***

b. $-2x + 150 = 0$ ***(M1)***

Note: Award ***(M1)*** for setting $P'(x) = 0$.

OR

Award ***(M1)*** for sketch of $P(x)$ and maximum point identified. ***(M1)***
$x = 75$ ***(A1)*(ft)** ***(C2)***

Note: Follow through from their answer to part (a).

c. $\frac{7875}{75}$ ***(M1)***

Note: Award ***(M1)*** for 7875 seen.

$= 105$ ***(A1)*(ft)** ***(C2)***

Note: Follow through from their answer to part (b).

Examiners report

a. [N/A]
b. [N/A]
c. [N/A]

The equation of line L_1 is $y = -\frac{2}{3}x - 2$.

Point P lies on L_1 and has x-coordinate -6.

The line L_2 is perpendicular to L_1 and intersects L_1 when $x = -6$.

a. Write down the gradient of L_1. [1]

b. Find the y-coordinate of P. [2]

c. Determine the equation of L_2. Give your answer in the form $ax + by + d = 0$, where a, b and d are integers. [3]

Markscheme

a. $-\frac{2}{3}$ ***(A1)*** ***(C1)***

[1 mark]

b. $y = -\frac{2}{3}(-6) - 2$ ***(M1)***

Note: Award ***(M1)*** for correctly substituting -6 into the formula for L_1.

$(y =)\ 2$ ***(A1)*** ***(C2)***

Note: Award ***(A0)(A1)*** for $(-6,\ 2)$ with or without working.

[2 marks]

c. gradient of L_2 is $\frac{3}{2}$ ***(A1)*****(ft)**

Note: Follow through from part (a).

$2 = \frac{3}{2}(-6) + c$ **OR** $y - 2 = \frac{3}{2}(x - (-6))$ ***(M1)***

Note: Award ***(M1)*** for substituting their part (b), their gradient and -6 into equation of a straight line.

$3x - 2y + 22 = 0$ ***(A1)*****(ft)** ***(C3)***

Note: Follow through from parts (a) and (b). Accept any integer multiple.

Award ***(A1)(M1)(A0)*** for $y = \frac{3}{2}x + 11$.

[3 marks]

Examiners report

a. [N/A]

b. [N/A]

c. [N/A]

a. Expand the expression $x(2x^3 - 1)$. [2]

b. Differentiate $f(x) = x(2x^3 - 1)$. [2]

c. Find the x-coordinate of the local minimum of the curve $y = f(x)$. [2]

Markscheme

a. $2x^4 - x$ ***(A1)(A1)*** ***(C2)***

Note: Award ***(A1)*** for $2x^4$, ***(A1)*** for $-x$.

[2 marks]

b. $8x^3 - 1$ ***(A1)*(ft)*(A1)*(ft)** ***(C2)***

Note: Award ***(A1)*(ft)** for $8x^3$, ***(A1)*(ft)** for -1. Follow through from their part (a).

Award at most ***(A1)(A0)*** if extra terms are seen.

[2 marks]

c. $8x^3 - 1 = 0$ ***(M1)***

Note: Award ***(M1)*** for equating their part (b) to zero.

$(x =)\frac{1}{2}$ (0.5) ***(A1)*(ft)** ***(C2)***

Notes: Follow through from part (b).

0.499 is the answer from the use of trace on the GDC; award ***(A0)(A0)***.

For an answer of $(0.5, -0.375)$, award ***(M1)(A0)***.

[2 marks]

Examiners report

a. A surprising number of candidates were unable to correctly expand the expression given in part (a). Most candidates were able to differentiate their function but a considerable number were unable to find the x-coordinate of the minimum point. Candidates must read the questions correctly as answers giving ordered pairs were not awarded the final mark. A number of candidates did not use calculus to determine the local minimum but graphed the function, often achieving full marks for part (c), even when parts (b) or (a) were incorrect or left blank.

b. A surprising number of candidates were unable to correctly expand the expression given in part (a). Most candidates were able to differentiate their function but a considerable number were unable to find the x-coordinate of the minimum point. Candidates must read the questions correctly as answers giving ordered pairs were not awarded the final mark. A number of candidates did not use calculus to determine the local minimum but graphed the function, often achieving full marks for part (c), even when parts (b) or (a) were incorrect or left blank.

c. A surprising number of candidates were unable to correctly expand the expression given in part (a). Most candidates were able to differentiate their function but a considerable number were unable to find the x-coordinate of the minimum point. Candidates must read the questions correctly as answers giving ordered pairs were not awarded the final mark. A number of candidates did not use calculus to determine the local minimum but graphed the function, often achieving full marks for part (c), even when parts (b) or (a) were incorrect or left blank.

---

a. Consider the function $f(x) = ax^2 + c$. [1]

Find $f'(x)$

b. Point $\mathrm{A}(-2,\ 5)$ lies on the graph of $y = f(x)$. The gradient of the tangent to this graph at A is -6 . [3]

Find the value of a .

c. Find the value of c . [2]

Markscheme

a. $2ax$ ***(A1) (C1)***

Note: Award ***(A1)*** for $2ax$. Award ***(A0)*** if other terms are seen.

b. $2a(-2) = -6$ ***(M1)(M1)***

Note: Award ***(M1)*** for correct substitution of $x = -2$ in their gradient function, ***(M1)*** for equating their gradient function to -6 . Follow through from part (a).

$(a =)1.5 \ \left(\frac{3}{2}\right)$ ***(A1)*(ft) *(C3)***

c. $\text{their } 1.5 \times (-2)^2 + c = 5$ ***(M1)***

Note: Award ***(M1)*** for correct substitution of their a and point A. Follow through from part (b).

$(c =) - 1$ ***(A1)*(ft) *(C2)***

Examiners report

a. Question 11: Equation of tangent

Part (a) was generally well answered.

b. In part (b), many candidates substituted the value of the function, rather than its gradient; this was usually correctly followed through into part (c).

c. In part (b), many candidates substituted the value of the function, rather than its gradient; this was usually correctly followed through into part (c).

Maria owns a cheese factory. The amount of cheese, in kilograms, Maria sells in one week, Q, is given by

$$Q = 882 - 45p,$$

where p is the price of a kilogram of cheese in euros (EUR).

Maria earns $(p - 6.80)$ EUR for each kilogram of cheese sold.

To calculate her weekly profit W, in EUR, Maria multiplies the amount of cheese she sells by the amount she earns per kilogram.

a. Write down how many kilograms of cheese Maria sells in one week if the price of a kilogram of cheese is 8 EUR. [1]

b. Find how much Maria earns in one week, from selling cheese, if the price of a kilogram of cheese is 8 EUR. [2]

c. Write down an expression for W in terms of p. [1]

d. Find the price, p, that will give Maria the highest weekly profit. [2]

Markscheme

a. 522 (kg) ***(A1)*** ***(C1)***

[1 mark]

b. $522(8 - 6.80)$ or equivalent ***(M1)***

Note: Award ***(M1)*** for multiplying their answer to part (a) by $(8 - 6.80)$.

626 (EUR) (626.40) ***(A1)*(ft)** ***(C2)***

Note: Follow through from part (a).

[2 marks]

c. $(W =)\ (882 - 45p)(p - 6.80)$ ***(A1)***

OR

$(W =) - 45p^2 + 1188p - 5997.6$ ***(A1)*** ***(C1)***

[1 mark]

d. sketch of W with some indication of the maximum ***(M1)***

OR

$-90p + 1188 = 0$ ***(M1)***

Note: Award ***(M1)*** for equating the correct derivative of their part (c) to zero.

OR

$(p=)\ \frac{-1188}{2\times(-45)}$ ***(M1)***

Note: Award ***(M1)*** for correct substitution into the formula for axis of symmetry.

$(p=)\ 13.2\ (\text{EUR})$ ***(A1)*(ft)** ***(C2)***

Note: Follow through from their part (c), if the value of p is such that $6.80 < p < 19.6$.

[2 marks]

Examiners report

a. [N/A]
b. [N/A]
c. [N/A]
d. [N/A]

Consider the curve $y = x^3 + kx$.

a. Write down $\frac{\mathrm{d}y}{\mathrm{d}x}$. [1]

b. The curve has a local minimum at the point where $x = 2$. [3]

Find the value of k.

c. The curve has a local minimum at the point where $x = 2$. [2]

Find the value of y at this local minimum.

Markscheme

a. $3x^2 + k$ ***(A1)*** ***(C1)***

[1 mark]

b. $3(2)^2 + k = 0$ ***(A1)*(ft)*(M1)***

Note: Award ***(A1)*(ft)** for substituting 2 in their $\frac{\mathrm{d}y}{\mathrm{d}x}$, ***(M1)*** for setting their $\frac{\mathrm{d}y}{\mathrm{d}x} = 0$.

$k = -12$ ***(A1)*(ft)** ***(C3)***

Note: Follow through from their derivative in part (a).

[3 marks]

c. $2^3 - 12 \times 2$ ***(M1)***

Note: Award ***(M1)*** for substituting 2 and their –12 into equation of the curve.

$= -16$ ***(A1)*(ft)** ***(C12)***

Note: Follow through from their value of k found in part (b).

[2 marks]

Examiners report

a. [N/A]
b. [N/A]
c. [N/A]

$f(x) = 5x^3 - 4x^2 + x$

a. Find $f'(x)$. [3]

b. Find using your answer to part (a) the *x*-coordinate of [3]

(i) the local maximum point;

(ii) the local minimum point.

Markscheme

a. $15x^2 - 8x + 1$ ***(A1)(A1)(A1)*** ***(C3)***

Note: Award ***(A1)*** for each correct term.

[3 marks]

b. $15x^2 - 8x + 1 = 0$ ***(A1)*(ft)**

Note: Award ***(A1)*(ft)** for setting their derivative to zero.

(i) $(x =)\frac{1}{5}(0.2)$ ***(A1)*(ft)**

(ii) $(x =)\frac{1}{3}(0.333)$ ***(A1)*(ft)** ***(C3)***

Notes: Follow through from their answer to part (a).

[3 marks]

Examiners report

a. Many candidates lost 1 mark in part (a) through not realizing that the derivative of x is 1. As a consequence, $15x^2 - 8x$ proved to be a popular answer.

b. Very few candidates gained the marks in part (b) to find the maximum and minimum point. Although the question indicated to use their answer to part (a), very few candidates set the derivative to zero which would have given them 1 mark. It seemed as if many candidates were trying to use their calculators to find the coordinates but could not find which was the maximum and which was the minimum.

---

a. The equation of the straight line L_1 is $y = 2x - 3.$ [1]

Write down the y-intercept of L_1 .

b. Write down the gradient of L_1 . [1]

c. The line L_2 is parallel to L_1 and passes through the point $(0,\ 3)$. [1]

Write down the equation of L_2 .

d. The line L_3 is perpendicular to L_1 and passes through the point $(-2,\ 6)$. [1]

Write down the gradient of L_3.

e. Find the equation of L_3 . Give your answer in the form $ax + by + d = 0$, where a , b and d are integers. [2]

Markscheme

a. $(0,\ -3)$ ***(A1) (C1)***

Note: Accept -3 or $y = -3$.

b. 2 ***(A1) (C1)***

c. $y = 2x + 3$ ***(A1)*(ft) *(C1)***

Note: Award ***(A1)*(ft)** for correct equation. Follow through from part (b)
Award ***(A0)*** for $L_2 = 2x + 3$.

d. $-\frac{1}{2}$ ***(A1)*(ft) *(C1)***

Note: Follow through from part (b).

e. $6 = -\frac{1}{2}(-2) + c$ ***(M1)***

$c = 5$ (may be implied)

OR

$y - 6 = -\frac{1}{2}(x + 2)$ ***(M1)***

Note: Award ***(M1)*** for correct substitution of their gradient in part (d) and the point $(-2,\ 6)$. Follow through from part (d).

$x + 2y - 10 = 0$ (or any integer multiple) ***(A1)*(ft)** ***(C2)***

Note: Follow through from (d). The answer must be in the form $ax + by + d = 0$ for the ***(A1)*(ft)** to be awarded. Accept any integer multiple.

Examiners report

a. Question 12: Linear function.

Many candidates demonstrated a good understanding of linear functions so successfully found the y-intercepts, gradient and equation in the form $y = mx + c$. However only the very best were able to rewrite this in the form $ax + by + d = 0$ where a, b and d are integers.

b. Question 12: Linear function.

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c. Question 12: Linear function.

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e. Question 12: Linear function.

Many candidates demonstrated a good understanding of linear functions so successfully found the y-intercepts, gradient and equation in the form $y = mx + c$. However only the very best were able to rewrite this in the form $ax + by + d = 0$ where a, b and d are integers.

---

a. A company sells fruit juices in cylindrical cans, each of which has a volume of $340\ \text{cm}^3$. The surface area of a can is $A\ \text{cm}^2$ and is given by the formula [3]

$A = 2\pi r^2 + \frac{680}{r}$,

where r is the radius of the can, in cm.

To reduce the cost of a can, its surface area must be minimized.

Find $\frac{\mathrm{d}A}{\mathrm{d}r}$

b. Calculate the value of r that minimizes the surface area of a can. [3]

Markscheme

a. $\left(\frac{\mathrm{d}A}{\mathrm{d}r}\right) = 4\pi r - \frac{680}{r^2}$ ***(A1)(A1)(A1)*** ***(C3)***

Note: Award ***(A1)*** for $4\pi r$ (accept $12.6r$), ***(A1)*** for -680, ***(A1)*** for $\frac{1}{r^2}$ or r^{-2}

Award at most ***(A1)(A1)(A0)*** if additional terms are seen.

b. $4\pi r - \frac{680}{r^2} = 0$ ***(M1)***

Note: Award ***(M1)*** for equating their $\frac{\mathrm{d}A}{\mathrm{d}r}$ to zero.

$4\pi r^3 - 680 = 0$ ***(M1)***

Note: Award ***(M1)*** for initial correct rearrangement of the equation. This may be assumed if $r^3 = \frac{680}{4\pi}$ or $r = \sqrt[3]{\frac{680}{4\pi}}$ seen.

OR

sketch of A with some indication of minimum point ***(M1)(M1)***

Note: Award ***(M1)*** for sketch of A, ***(M1)*** for indication of minimum point.

OR

sketch of $\frac{\mathrm{d}A}{\mathrm{d}r}$ with some indication of zero ***(M1)(M1)***

Note: Award ***(M1)*** for sketch of $\frac{\mathrm{d}A}{\mathrm{d}r}$, ***(M1)*** for indication of zero.

$(r =)\ 3.78\ (\mathrm{cm}) \quad (3.78239...)$ ***(A1)*(ft) *(C3)***

Note: Follow through from part (a).

Examiners report

a. Question 15: Optimization

Many candidates were able to differentiate in part (a), but then were unable to relate this to part (b). However, it seemed that many more had not studied the calculus at all.

b. Question 15: Optimization

Many candidates were able to differentiate in part (a), but then were unable to relate this to part (b). However, it seemed that many more had not studied the calculus at all.

Consider the function $f(x) = 2x^3 - 5x^2 + 3x + 1$.

a. Find $f'(x)$. [3]

b. Write down the value of $f'(2)$. [1]

c. Find the equation of the tangent to the curve of $y = f(x)$ at the point $(2, 3)$. [2]

Markscheme

a. $f'(x) = 6x^2 - 10x + 3$ ***(A1)(A1)(A1)*** ***(C3)***

Notes: Award ***(A1)*** for each correct term and no extra terms.
Award ***(A1)(A1)(A0)*** if each term correct and extra term seen.
Award***(A1)(A0)(A0)*** if two terms correct and extra term seen.
Award ***(A0)*** otherwise.

[3 marks]

b. $f'(2) = 7$ ***(A1)*(ft)** ***(C1)***

[1 mark]

c. $y = 7x - 11$ or equivalent ***(A1)*(ft)*(A1)*(ft)** ***(C2)***

Note: Award ***(A1)*(ft)** on their (b) for $7x$ (must have x), ***(A1)*(ft)** for -11. Accept $y - 3 = 7(x - 2)$.

[2 marks]

Examiners report

a. Most candidates were able to score full marks for parts (a) and (b). When mistakes were made in part (a) follow-through marks could be awarded for part (b) provided working was shown. Part (c) was disappointing with many candidates not realizing that the answer in (b) was the gradient of the tangent line.

b. Most candidates were able to score full marks for parts (a) and (b). When mistakes were made in part (a) follow-through marks could be awarded for part (b) provided working was shown. Part (c) was disappointing with many candidates not realizing that the answer in (b) was the gradient of the tangent line.

c. Most candidates were able to score full marks for parts (a) and (b). When mistakes were made in part (a) follow-through marks could be awarded for part (b) provided working was shown. Part (c) was disappointing with many candidates not realizing that the answer in (b) was the gradient of the tangent line.

The equation of a curve is $y = \frac{1}{2}x^4 - \frac{3}{2}x^2 + 7$.

The gradient of the tangent to the curve at a point P is -10.

a. Find $\frac{\mathrm{d}y}{\mathrm{d}x}$. [2]

b. Find the coordinates of P. [4]

Markscheme

a. $2x^3 - 3x$ ***(A1)(A1)*** ***(C2)***

Note: Award ***(A1)*** for $2x^3$, award ***(A1)*** for $-3x$.

Award at most ***(A1)(A0)*** if there are any extra terms.

[2 marks]

b. $2x^3 - 3x = -10$ ***(M1)***

Note: Award ***(M1)*** for equating their answer to part (a) to -10.

$x = -2$ ***(A1)*(ft)**

Note: Follow through from part (a). Award ***(M0)(A0)*** for -2 seen without working.

$y = \frac{1}{2}(-2)^4 - \frac{3}{2}(-2)^2 + 7$ ***(M1)***

Note: Award ***(M1)*** substituting their -2 into the original function.

$y = 9$ ***(A1)*(ft)** ***(C4)***

Note: Accept $(-2,\ 9)$.

[4 marks]

Examiners report

a. [N/A]
b. [N/A]

The table given below describes the behaviour of $f'(x)$, the derivative function of $f(x)$, in the domain $-4 < x < 2$.

x	$f'(x)$
$-4 < x < -2$	< 0
-2	0
$-2 < x < 1$	> 0
1	0
$1 < x < 2$	> 0

a. State whether *f* (0) is greater than, less than or equal to *f* (–2). Give a reason for your answer. [2]

b. The point P(–2, 3) lies on the graph of *f* (*x*). [2]

Write down the equation of the tangent to the graph of *f* (*x*) at the point P.

c. The point P(–2, 3) lies on the graph of *f* (*x*). [2]

From the information given about *f* ′(*x*), state whether the point (–2, 3) is a maximum, a minimum or neither. Give a reason for your answer.

Markscheme

a. greater than ***(A1)***

Gradient between *x* = –2 and *x* = 0 is positive. ***(R1)***

OR

The function is increased between these points or equivalent. ***(R1)*** ***(C2)***

Note: Accept a sketch. Do not award ***(A1)(R0)***.

[2 marks]

b. *y* = 3 ***(A1)(A1)*** ***(C2)***

Note: Award ***(A1)*** for *y* = a constant, ***(A1)*** for 3.

[2 marks]

c. minimum ***(A1)***

Gradient is negative to the left and positive to the right or equivalent. ***(R1)*** ***(C2)***

Note: Accept a sketch. Do not award ***(A1)(R0)***.

[2 marks]

Examiners report

a. Very few candidates received full marks for this question and many omitted the question completely. A sketch showing the information provided in the table would have been very useful but few candidates chose this approach.

b. Very few candidates received full marks for this question and many omitted the question completely. A sketch showing the information provided in the table would have been very useful but few candidates chose this approach.

c. Very few candidates received full marks for this question and many omitted the question completely. A sketch showing the information provided in the table would have been very useful but few candidates chose this approach.

The straight line, L, has equation $2y - 27x - 9 = 0$.

a. Find the gradient of *L*. [2]

b. Sarah wishes to draw the tangent to $f(x) = x^4$ parallel to *L*. [1]

Write down $f'(x)$.

c, i.Find the *x* coordinate of the point at which the tangent must be drawn. [2]

c, iiWrite down the value of $f(x)$ at this point. [1]

Markscheme

a. $y = 13.5x + 4.5$ ***(M1)***

Note: Award ***(M1)*** for $13.5x$ seen.

gradient = 13.5 ***(A1)*** ***(C2)***

[2 marks]

b. $4x^3$ ***(A1)*** ***(C1)***

[1 mark]

c, i. $4x^3 = 13.5$ ***(M1)***

Note: Award ***(M1)*** for equating their answers to (a) and (b).

$x = 1.5$ ***(A1)*(ft)**

[2 marks]

c, ii. $\frac{81}{16}$ (5.0625, 5.06) ***(A1)*(ft)** ***(C3)***

Note: Award ***(A1)*(ft)** for substitution of their (c)(i) into $x4$ with working seen.

[1 mark]

Examiners report

a. The structure of this question was not well understood by the majority; the links between parts not being made. Again, this question was included to discriminate at the grade 6/7 level.

Most were successful in this part.

b. The structure of this question was not well understood by the majority; the links between parts not being made. Again, this question was included to discriminate at the grade 6/7 level.

This part was usually well attempted.

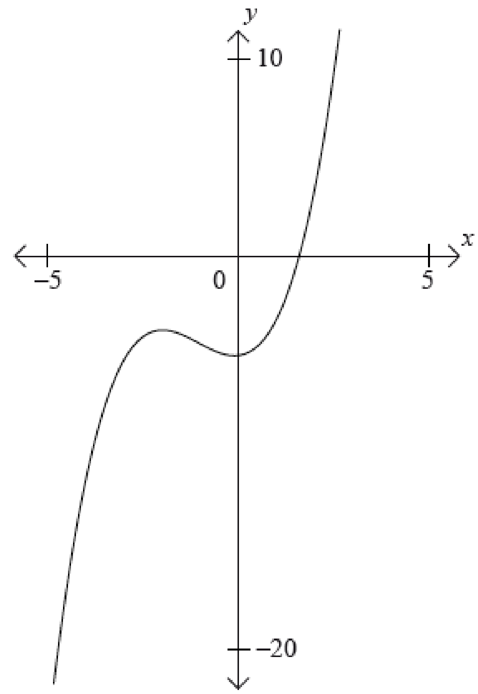
c, i. The structure of this question was not well understood by the majority; the links between parts not being made. Again, this question was included to discriminate at the grade 6/7 level.

Only the best candidates succeeded in this part.

c, ii. The structure of this question was not well understood by the majority; the links between parts not being made. Again, this question was included to discriminate at the grade 6/7 level.

Only the best candidates succeeded in this part.

Consider the graph of the function $f(x) = x^3 + 2x^2 - 5$.



a. Label the local maximum as A on the graph. [1]

b. Label the local minimum as B on the graph. [1]

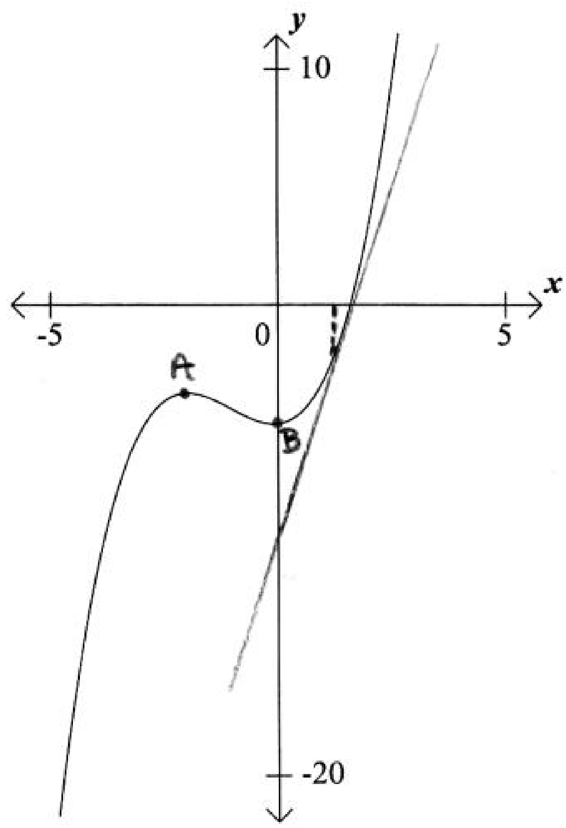
c. Write down the interval where $f'(x) < 0$. [1]

d. Draw the tangent to the curve at $x = 1$ on the graph. [1]

e. Write down the equation of the tangent at $x = 1$. [2]

Markscheme

a.



correct label on graph ***(A1)*** ***(C1)***

[1 mark]

b.

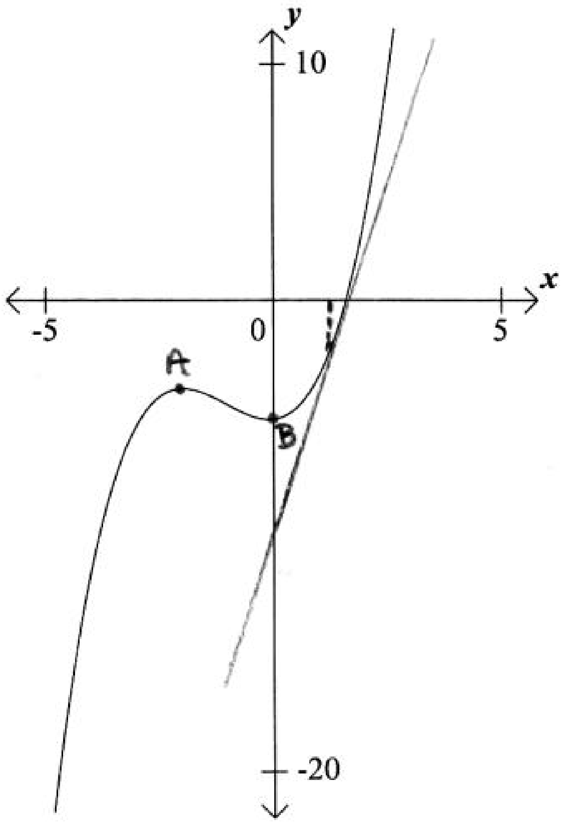
correct label on graph ***(A1)*** ***(C1)***

[1 mark]

c. $-1.33 < x < 0 \quad \left(-\frac{4}{3} < x < 0\right)$ ***(A1)*** ***(C1)***

[1 mark]

d.



tangent drawn at $x = 1$ on graph ***(A1)*** ***(C1)***

[1 mark]

e. $y = 7x - 9$ ***(A1)(A1)*** ***(C2)***

Notes: Award ***(A1)*** for 7, ***(A1)*** for -9.

If answer not given as an equation award at most ***(A1)(A0)***.

[2 marks]

Examiners report

a. [N/A]
b. [N/A]
c. [N/A]
d. [N/A]
e. [N/A]

A curve is described by the function $f(x) = 3x - \frac{2}{x^2}, x \neq 0$.

a. Find $f'(x)$. [3]

b. The gradient of the curve at point A is 35. [3]

Find the *x*-coordinate of point A.

Markscheme

a. $f'(x) = 3 + \frac{4}{x^3}$ ***(A1)(A1)(A1) (C3)***

Notes: Award ***(A1)*** for 3, ***(A1)*** for + 4 and ***(A1)*** for $\frac{1}{x^3}$ or x^{-3}. Award at most ***(A1)(A1)(A0)*** if additional terms are seen.

b. $3 + \frac{4}{x^3} = 35$ ***(M1)***

Note: Award ***(M1)*** for equating their derivative to 35 only if the derivative is **not** a constant.

$x^3 = \frac{1}{8}$ ***(A1)*****(ft)**

$\frac{1}{2}(0.5)$ ***(A1)*****(ft)** ***(C3)***

Examiners report

a. [N/A]
b. [N/A]

$$f(x) = \frac{1}{3}x^3 + 2x^2 - 12x + 3$$

a. Find $f'(x)$. [3]

b. Find the interval of x for which $f(x)$ is decreasing. [3]

Markscheme

a. $f'(x) = x^2 + 4x - 12$ ***(A1)(A1)(A1) (C3)***

Notes: Award ***(A1)*** for each term. Award at most ***(A1)(A1)(A0)*** if other terms are seen.

[3 marks]

b. $-6 \leqslant x \leqslant 2$ **OR** $-6 < x < 2$ ***(A1)*****(ft)*****(A1)*****(ft)*****(A1)*** ***(C3)***

Notes: Award ***(A1)*****(ft)** for -6, ***(A1)*****(ft)** for 2, ***(A1)*** for consistent use of strict (<) or weak (⩽) inequalities. Final ***(A1)*** for correct interval notation (accept alternative forms). This can only be awarded when the left hand side of the inequality is less than the right hand side of the inequality. Follow through from their solutions to their $f'(x) = 0$ only if working seen.

[3 marks]

Examiners report

a. This question was quite a good differentiator with many able to score at least one mark in part (a). Part (b) proved however to be quite a challenge as many candidates did not seem to understand what was required and were unable to use their answer to part (a) to help them to meet the demands of this question part. The top quartile scored well with virtually everyone scoring at least three marks. The picture was somewhat reversed with the lower quartile with the majority of candidates scoring 2 or fewer marks.

b. This question was quite a good differentiator with many able to score at least one mark in part (a). Part (b) proved however to be quite a challenge as many candidates did not seem to understand what was required and were unable to use their answer to part (a) to help them to meet the demands of this question part. The top quartile scored well with virtually everyone scoring at least three marks. The picture was somewhat reversed with the lower quartile with the majority of candidates scoring 2 or fewer marks.

Let $f(x) = x^4$.

a. Write down $f'(x)$. [1]

b. Point $\mathrm{P}(2, 6)$ lies on the graph of f. [2]

Find the gradient of the tangent to the graph of $y = f(x)$ at P.

c. Point $\mathrm{P}(2, 16)$ lies on the graph of f. [3]

Find the equation of the normal to the graph at P. Give your answer in the form $ax + by + d = 0$, where a, b and d are integers.

Markscheme

a. $(f'(x) =)\ \ 4x^3$ ***(A1) (C1)***

[1 mark]

b. 4×2^3 ***(M1)***

Note: Award ***(M1)*** for substituting 2 into their derivative.

$= 32$ ***(A1)*(ft)** ***(C2)***

Note: Follow through from their part (a).

[2 marks]

c. $y - 16 = -\frac{1}{32}(x - 2)$ **or** $y = -\frac{1}{32}x + \frac{257}{16}$ ***(M1)(M1)***

Note: Award ***(M1)*** for their gradient of the normal seen, ***(M1)*** for point substituted into equation of a straight line in only x and y (with any constant 'c' eliminated).

$x + 32y - 514 = 0$ **or** any integer multiple ***(A1)*(ft)*** ***(C3)***

Note: Follow through from their part (b).

[3 marks]

Examiners report

a. [N/A]
b. [N/A]
c. [N/A]

Consider $f : x \mapsto x^2 - 4$.

a. Find $f'(x)$. [1]

b. Let *L* be the line with equation *y* = 3*x* + 2. [1]

Write down the gradient of a line parallel to *L*.

c. Let *L* be the line with equation *y* = 3*x* + 2. [4]

Let P be a point on the curve of *f*. At P, the tangent to the curve is parallel to *L*. Find the coordinates of P.

Markscheme

a. $2x$ ***(A1)*** ***(C1)***

[1 mark]

b. 3 ***(A1)*** ***(C1)***

[1 mark]

c. $2x = 3$ ***(M1)***

Note: ***(M1)*** for equating their (a) to their (b).

$x = 1.5$ ***(A1)*(ft)**

$y = (1.5)^2 - 4$ ***(M1)***

Note: ***(M1)*** for substituting their *x* in *f* (*x*).

(1.5, −1.75) (accept *x* = 1.5, *y* = −1.75) ***(A1)*(ft)** ***(C4)***

Note: Missing coordinate brackets receive ***(A0)*** if this is the first time it occurs.

[4 marks]

Examiners report

a. This question was generally answered well in parts (a) and (b).

b. This question was generally answered well in parts (a) and (b).

c. This part proved to be difficult as candidates did not realise that to find the value of the x coordinate they needed to equate their answers to the first two parts. They did not understand that the first derivative is the gradient of the function. Some found the value of x, but did not substitute it back into the function to find the value of y.

A function f is given by $f(x) = 4x^3 + \frac{3}{x^2} - 3,\ x \neq 0$.

a. Write down the derivative of f. [3]

b. Find the point on the graph of f at which the gradient of the tangent is equal to 6. [3]

Markscheme

a. $12x^2 - \frac{6}{x^3}$ or equivalent ***(A1)(A1)(A1)*** ***(C3)***

Note: Award ***(A1)*** for $12x^2$, ***(A1)*** for -6 and ***(A1)*** for $\frac{1}{x^3}$ or x^{-3}. Award at most ***(A1)(A1)(A0)*** if additional terms seen.

[3 marks]

b. $12x^2 - \frac{6}{x^3} = 6$ ***(M1)***

Note: Award ***(M1)*** for equating their derivative to 6.

$(1,\ 4)$ **OR** $x = 1,\ y = 4$ ***(A1)*(ft)*(A1)*(ft)** ***(C3)***

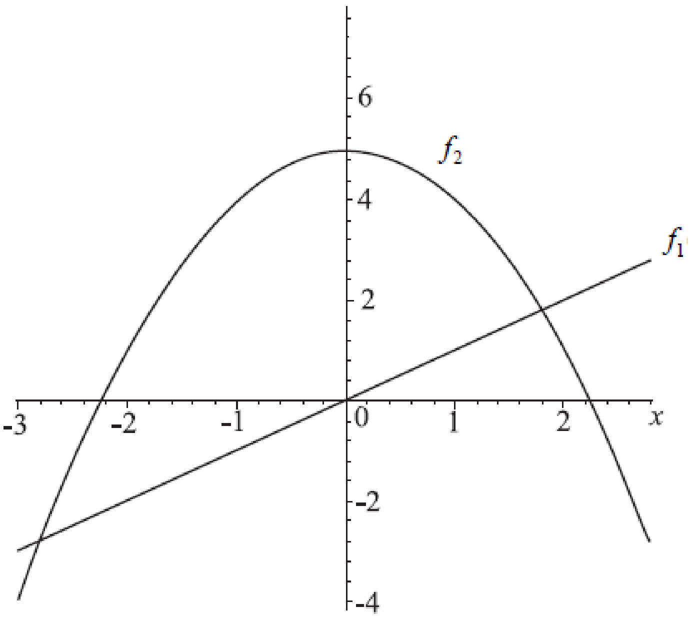
Note: A frequent wrong answer seen in scripts is $(1,\ 6)$ for this answer with correct working award ***(M1)(A0)(A1)*** and if there is no working award ***(C1)***.

[3 marks]

Examiners report

a. [N/A]
b. [N/A]

The figure below shows the graphs of functions $f_1(x) = x$ and $f_2(x) = 5 - x^2$.



a. (i) Differentiate $f_1(x)$ with respect to *x*. [3]

(ii) Differentiate $f_2(x)$ with respect to *x*.

b. Calculate the value of *x* for which the gradient of the two graphs is the same. [2]

c. Draw the tangent to the **curved** graph for this value of *x* on the figure, showing clearly the property in part (b). [1]

Markscheme

a. (i) $f_1'(x) = 1$ ***(A1)***

(ii) $f_2'(x) = -2x$ ***(A1)(A1)***

(A1) *for correct differentiation of each term.* ***(C3)***

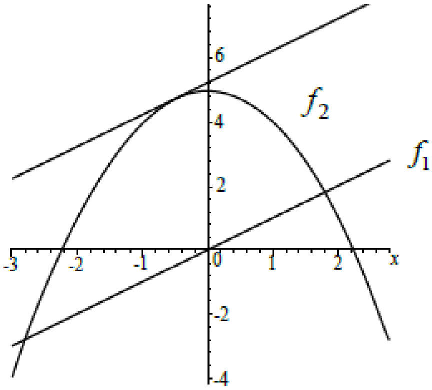
[3 marks]

b. $1 = -2x$ ***(M1)***

$x = -\frac{1}{2}$ ***(A1)*****(ft)** ***(C2)***

[2 marks]

c. ***(A1)*** *is for the tangent drawn at* $x = \frac{1}{2}$ *and reasonably parallel to the line* f_1 *as shown.*



(A1) (C1)

[1 mark]

Examiners report

a. Most candidates were able to differentiate correctly, but only a third were able to calculate the value of *x* for which the gradients of the graphs were the same and a similar number did not attempt to. Some found the *x*-coordinate of the point of intersection.

b. Most candidates were able to differentiate correctly, but only a third were able to calculate the value of *x* for which the gradients of the graphs were the same and a similar number did not attempt to. Some found the *x*-coordinate of the point of intersection.

c. c) Very few candidates were able to draw the tangent correctly. Some tangents were drawn horizontally and some at the point of intersection. The line could have been drawn without any knowledge of calculus so the indication here was that many of the candidates misunderstood the question.

A function is given as $f(x) = 2x^3 - 5x + \frac{4}{x} + 3, \ -5 \leqslant x \leqslant 10, \ x \neq 0.$

a. Write down the derivative of the function. [4]

b. Use your graphic display calculator to find the coordinates of the local minimum point of $f(x)$ in the given domain. [2]

Markscheme

a. $6x^2 - 5 - \frac{4}{x^2}$ ***(A1)(A1)(A1)(A1)*** ***(C4)***

Note: Award ***(A1)*** for $6x^2$, ***(A1)*** for -5, ***(A1)*** for -4, ***(A1)*** for x^{-2} or $\frac{1}{x^2}$.

Award at most ***(A1)(A1)(A1)(A0)*** if additional terms are seen.

[4 marks]

b. $(1.15, \ 3.77)$ $((1.15469\ldots, 3.76980\ldots))$ ***(A1)(A1)*** ***(C2)***

Notes: Award ***(A1)(A1)*** for "$x = 1.15$ and $y = 3.77$".

Award at most ***(A0)(A1)*(ft)** if parentheses are omitted.

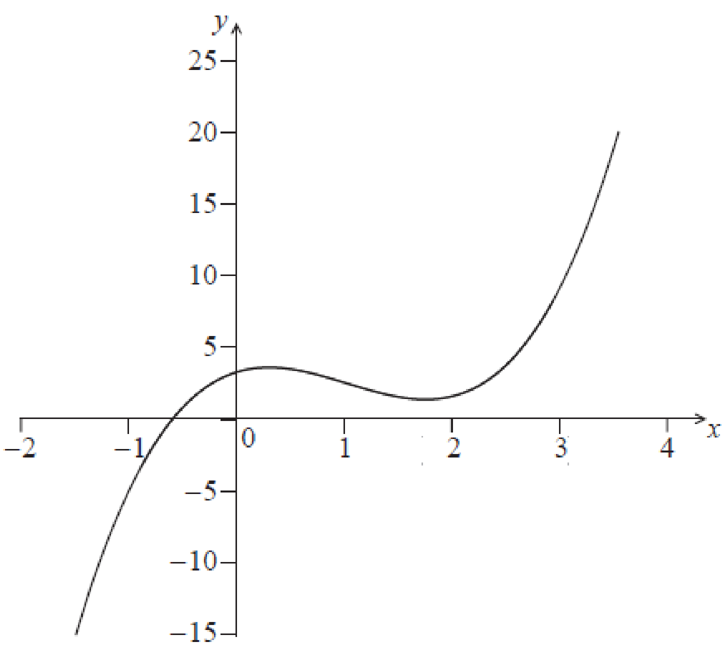
[2 marks]

Examiners report

a. [N/A]
b. [N/A]

---

a. Consider the function $f(x) = x^3 - 3x^2 + 2x + 2$. Part of the graph of f is shown below. [3]



Find $f'(x)$.

b. There are two points at which the gradient of the graph of f is 11. Find the x-coordinates of these points. [3]

Markscheme

a. $(f'(x) =)\ 3x^2 - 6x + 2$ ***(A1)(A1)(A1)*** ***(C3)***

Note: Award ***(A1)*** for $3x^2$, ***(A1)*** for $-6x$ and ***(A1)*** for $+2$.
Award at most ***(A1)(A1)(A0)*** if there are extra terms present.

b. $11 = 3x^2 - 6x + 2$ ***(M1)***

Note: Award ***(M1)*** for equating their answer from part (a) to 11, this may be implied from $0 = 3x^2 - 6x - 9$.

$(x =)\ -1\ ,\ \ (x =)\ 3$ ***(A1)*(ft)*(A1)*(ft)** ***(C3)***

Note: Follow through from part (a).
If final answer is given as coordinates, award at most ***(M1)(A0)(A1)*(ft)** for $(-1,\ -4)$ and $(3,\ 8)$.

Examiners report

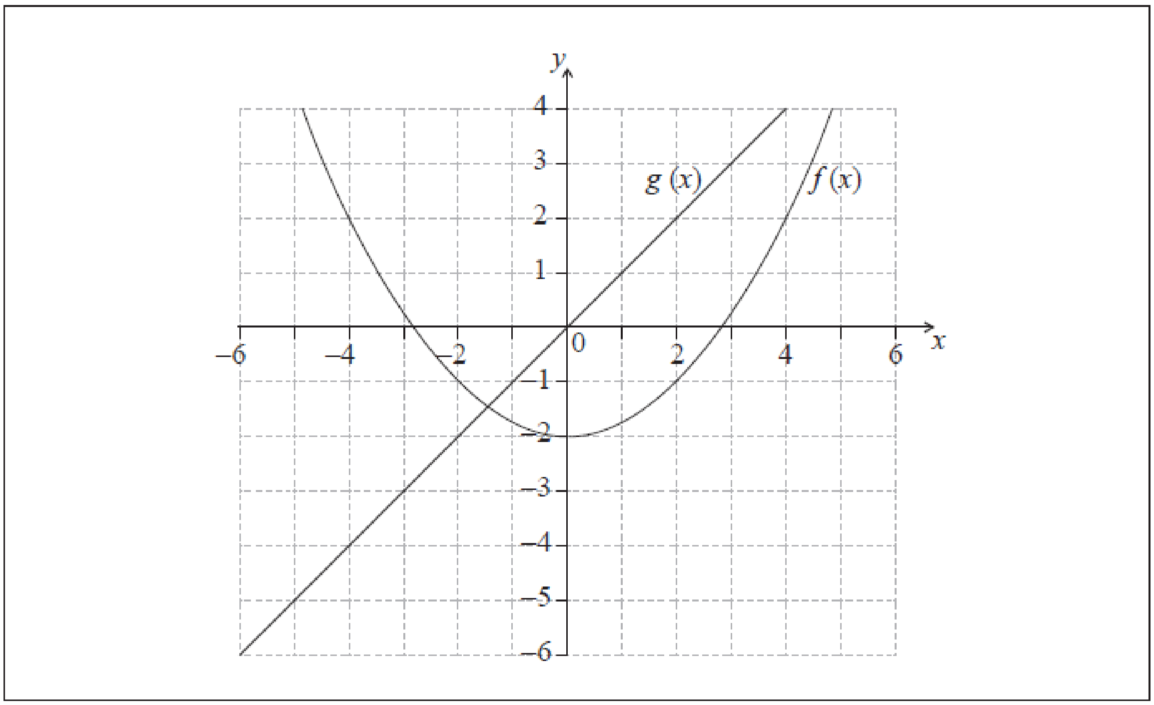
a. Question 15: Differential calculus.

Many candidates correctly differentiated the cubic equation. Most candidates were unable to use differential calculus to find the point where a cubic function had a specified gradient.

b. Question 15: Differential calculus.

Many candidates correctly differentiated the cubic equation. Most candidates were unable to use differential calculus to find the point where a cubic function had a specified gradient.

The figure shows the graphs of the functions $f(x) = \frac{1}{4}x^2 - 2$ and $g(x) = x$.



a. Differentiate $f(x)$ with respect to x . [1]

b. Differentiate $g(x)$ with respect to x . [1]

c. Calculate the value of x for which the gradients of the two graphs are the same. [2]

d. Draw the tangent to the parabola at the point with the value of x found in part (c). [2]

Markscheme

a. $\frac{1}{2}x\ \left(\frac{2}{4}x\right)$ ***(A1) (C1)***

Note: Accept an equivalent, unsimplified expression (i.e. $2 \times \frac{1}{4}x$).

[1 mark]

b. 1 ***(A1) (C1)***

[1 mark]

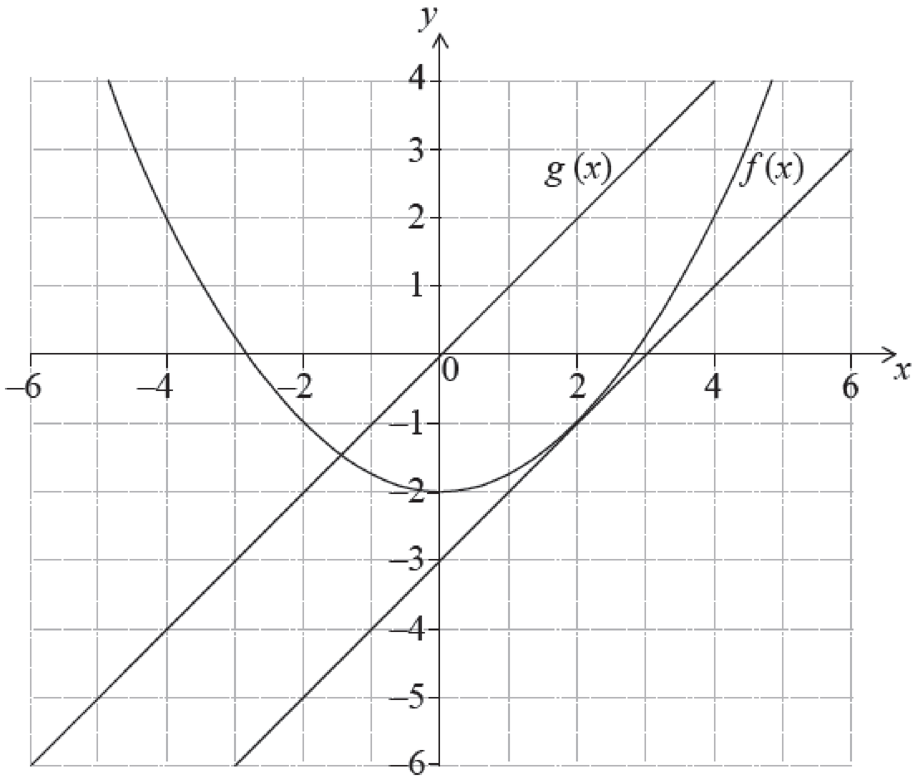
c. $\frac{1}{2}x = 1$ ***(M1)***

$x = 2$ ***(A1)*****(ft)** ***(C2)***

Notes: Award ***(M1)(A0)*** for coordinate pair $(2,\ -1)$ seen with or without working. Follow through from their answers to parts (a) and (b).

[2 marks]

d.



tangent drawn to the parabola at the x-coordinate found in part (c) ***(A1)*****(ft)**

candidate's attempted tangent drawn parallel to the graph of $g(x)$ ***(A1)*****(ft)** ***(C2)***

[2 marks]

Examiners report

a. Parts (a) and (b) were reasonably well attempted indicating that candidates are well drilled in the process of differentiation. Correct answers however in part (c) proved elusive to many as frequent attempts to equate the two given functions rather than the gradients of the given functions resulted in a popular, but incorrect, answer of $x = -1.46$. Part (d) was poorly attempted with many candidates simply either not attempting to draw a tangent or drawing it in the wrong place.

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A factory produces shirts. The cost, *C*, in Fijian dollars (FJD), of producing *x* shirts can be modelled by

$$C(x) = (x - 75)^2 + 100.$$

The cost of production should not exceed 500 FJD. To do this the factory needs to produce at least 55 shirts and at most *s* shirts.

a. Find the cost of producing 70 shirts. [2]

b. Find the value of *s*. [2]

c. Find the number of shirts produced when the cost of production is lowest. [2]

Markscheme

a. $(70 - 75)^2 + 100$ ***(M1)***

Note: Award ***(M1)*** for substituting in $x = 70$.

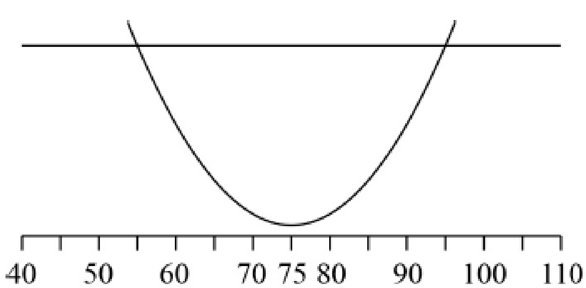
125 ***(A1) (C2)***

[2 marks]

b. $(s - 75)^2 + 100 = 500$ ***(M1)***

Note: Award ***(M1)*** for equating $C(x)$ to 500. Accept an inequality instead of =.

OR

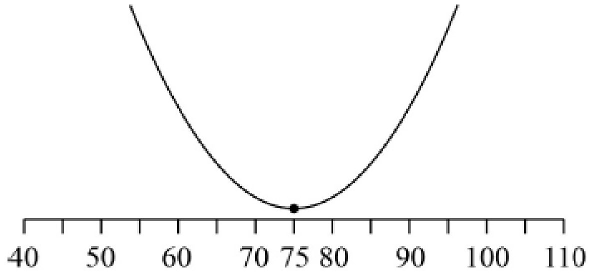


(M1)

Note: Award ***(M1)*** for sketching correct graph(s).

$(s =)$ 95 ***(A1) (C2)***

[2 marks]

c. 

(M1)

Note: Award ***(M1)*** for an attempt at finding the minimum point using graph.

OR

$\frac{95+55}{2}$ ***(M1)***

Note: Award ***(M1)*** for attempting to find the mid-point between their part (b) and 55.

OR

$(C'(x) =)\ 2x - 150 = 0$ ***(M1)***

Note: Award ***(M1)*** for an attempt at differentiation that is correctly equated to zero.

75 ***(A1) (C2)***

[2 marks]

Examiners report

a. [N/A]
b. [N/A]
c. [N/A]